APPENDIX A

ELASTICITY SOLUTION FOR A RING SEGMENT

A ring segment is shown in Figure 29. Its geometry is defined by the radii r_1 and r_2 and the angle α. The loading of the segment is a pressure p_1 at r_1 and p_2 at r_2. For equilibrium, p_2 is related to p_1 by Equation (24) in the text; i.e.,

$$p_2 = \frac{p_1}{k_2} \tag{A.1}$$

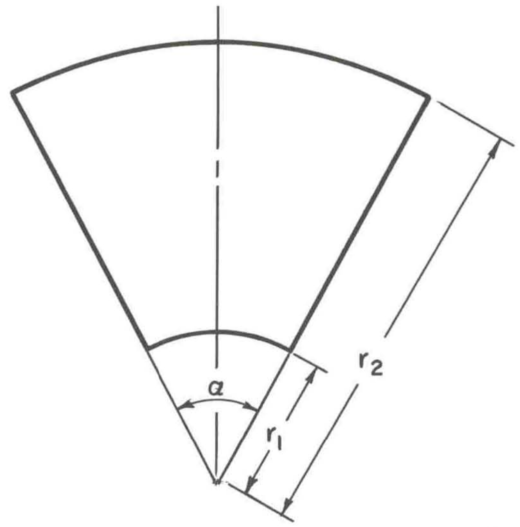

FIGURE 29. GEOMETRY OF RING SEGMENT

The solution for the stresses within the segment is found by superposition of two solutions: The Lamé solution for a cylinder, Equations (16a-c) and (17a, b) in the text, plus a bending solution, Equations (48) and (53) in Reference (19). The bending solution removes the moment from the sides of the segment that exists in the Lamé solution. The latter equations for the bending solution are written as

$$(\sigma_r)_b = \frac{4M_1p_1}{\beta_1} f_1(r), \; (\sigma_\theta)_b = \frac{4M_1p_1}{\beta_1} f_2(r), \; (\tau_{r\theta})_b = 0 \tag{A.2a-c}$$

and

$$\left(\frac{u}{r}\right)_b = \frac{M_1 p_1}{E_2 \beta_1} f_3(r) + \frac{G_1 p_1}{r} \cos\theta$$

(A. 3a-c)

$$\left(\frac{v}{r}\right)_b = \frac{8M_1 p_1}{E_2 \beta_1} (k_2^2 - 1)\,\theta - \frac{G_1 p_1}{r} \sin\theta$$

where $f_1(r)$, $f_2(r)$, and $f_3(r)$ are defined by Equations (23a-c) in the text and where

$$\beta_1 \equiv (k_2^2 - 1)^2 - 4k_2^2 (\log k_2)^2 \qquad \text{(A. 4)}$$

The moment $M = M_1 p_1 r_1^2$ is found by integrating the negative of the Lamé hoop stress $(\sigma_\theta)_c$ for a cylinder given by Equation (16b) in the text over the side of the segment; i.e.,

$$M = -\int_{r_1}^{r_2} (\sigma_\theta)_c \, r \, dr \quad ,$$

hence,

$$M_1 = \frac{-1}{p_1 r_1^2} \int_{r_1}^{r_2} \left\{ \frac{(p_1 - p_2 k_2^2)}{k_2^2 - 1} - \frac{(p_2 - p_1)\, k_2^2}{k_2^2 - 1} \left(\frac{r_1}{r}\right)^2 \right\} r \, dr$$

$$M_1 = -\frac{1}{2}\left(1 - \frac{p_2}{p_1} k_2^2\right) + \left(\frac{p_2}{p_1} - 1\right) \frac{k_2^2}{k_2^2 - 1} \log k_2 \qquad \text{(A. 5)}$$

G_1 is found by taking a reference point for the radial deflection u. If the point $r_0 = \dfrac{r_1 + r_2}{2}$, $\theta = 0$ is fixed,

then

$$G_1 = -\frac{M_1 r_0}{E_2 \beta_1} \left\{ -4\,(1 + v)\, k_2^2 \left(\frac{r_1}{r_0}\right)^2 \log k_2 + 4(1-v) \left[k_2^2 \log\left(\frac{r_0}{r_1}\right) - \log \frac{r_0}{r_1} \right] - 4(k_2^2 - 1) \right\} \qquad \text{(A. 6)}$$

The equations for the total stresses and displacements in ring segments were programmed on the computer and some calculations carried out. Example results are given in Table 14 for $k_2 = 2.0$ and $\alpha = 60$ degrees. It is noted that a small residual stress σ_θ remains on the side of the segments. To be more accurate, i.e., to achieve sides entirely free of stress, the residual σ_θ could be removed using a "dipole" solution in addition to the bending solution. However, the self-equilibrating residual stress that would be removed has a local edge effect according to the principle of St. Venant. Therefore, the σ_θ stresses in Table 14 are believed to be indicative of the actual magnitude of hoop stresses in segments at the center.